CONVERSIONS IN THE METRIC SYSTEM: REFLECTING ON THE GET MATHEMATICS CURRICULUM

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In this paper I argue that a fragmented approach to the teaching of the SI (Standard International) units – as dealt with in the National Curriculum Statement/ Curriculum and Assessment Policy Statement - Intermediate and Senior Phase Mathematics as part of General Education and Training – (GET) - when compared to the decimal system (with base 10), may interfere with understanding and conceptualisation, and with solving problems involving conversion for both learners and teachers. I also argue that isolating and fragmenting SI measurement units, complicate processes of accommodation and assimilation as advocated by Piaget. In terms of the Gestalt learning theory, fragmentation (in this context) prevents learners from seeing the greater meaning of the relationship(s) that exist among the different SI units. It thus appears, as if the curriculum writers are in favour of a more instrumental approach to parts of the curriculum as opposed to relational, which essentially also focus on the underlying mathematical relationships to enhance sense-making.

INTRODUCTION

If anybody in the street were to be asked to express an opinion on the position, role or importance of measurement in the Intermediate and Senior Phase mathematics curriculum (as currently used within the National Curriculum Statement in South Africa), such a person would most probably recognize and appreciate its value in terms of everyday requirements; economically, technologically, and socially. Yet, the actual time allotted to measurement in the mathematics classroom, let alone whether this is time spent effectively and efficiently, needs to be considered.

This opinion paper critiques the fragmented and instrumental approach to metric conversion in the NCS for the GET (which refers to the General Education and Training band, from Grade R to grade 9). I argue that fragmentation of the SI (Standard International) units, especially, when compared to the decimal system (with base 10), interferes with understanding and effectively dealing with problems involving conversion for both learners and teachers. I also argue that isolating and fragmenting SI measurement units, complicate processes of accommodation and assimilation as advocated by Piaget (1968). Furthermore, that in terms of the Gestalt learning theory fragmentation prevents learners from seeing the "greater meaning" (Dabba, 1999, p. 1) of the relationships that exist (Bragg & Outhred, 2004) between the different SI units. Thirdly, instrumental learning as opposed to relational learning, reinforces rote learning and the use of rules without understanding the underlying mathematical concepts (Skemp, 2006, p. 88).

My experiences as an in-service trainer of teachers at CTLI (Cape Teaching and Leadership Institute, Western Cape Education Department) since 2005 had given me the impression that measurement or mensuration as part of the intermediate and senior phase mathematics seemed to be problematic. Several years of facilitation at schools in the Western Cape, and observing learners doing the CTA (Common Task for Assessment – an externally set assessment written towards the end of the Senior Phase, that is, grade 9) showed that most of them could not measure accurately, and had limited knowledge about the relationships between units of length for instance. The CTLI is the official training arm of the Western Cape Education Department and runs in-service programs for Foundation, Intermediate and Senior Phase teachers. These teachers attend four-week courses (as fortnightly sessions) on a full-time basis, but only if a substitute is appointed for the duration of the course.

Instead of allowing learners to be confronted by the bigger whole (from kilo (10^3) to milli (10^{-3}), curriculum developers seem to prejudge Intermediate and Senior Phase learners' cognitive abilities by anticipating that they would not be mentally ready to adequately comprehend or meaningfully deal with the concepts mentioned. These concepts refer to the meaning of and relationships between units of length, units of capacity or units of mass.

Rationale for pursuing this topic

Based on my experiences in mathematics classrooms while observing lessons, the effectiveness and efficiency of methodologies dealing with measurement, compelled me to look at the situation analytically. According to (Kamii and Clark, 1997, p. 120) "typical instruction treats measurement as a mere empirical procedure rather than ... requiring reasoning". Consequently, Osborne's (1980, p. 54-68) critical analysis of the role of measurement in the mathematics classroom deserves to be looked at closely.

Osborne (1980, p. 55) emphasises the need for effective organization of teaching strategies to focus primarily on the "fundamental characteristics of measurement" that forms the basis of any measurement system. He believes this would allow the learner "to eventually transfer what was learned from one system of measurement to another". At the same time, however, measurement ideas and concepts should be applied to facilitate the development of numerical skills and understanding of related concepts. Teaching programs and lessons on measurement are not effectively geared towards acquiring the crucial concepts of measurement (Hiebert, 1984; Battista, 2006). Osborne (1980, p. 56) maintains that learning activities related to measurement generally do not "help learners form a sufficiently powerful ideation structure to enable them to deal successfully with new learning and problem solving involving measurement". Kamii and Clark (1997, p. 121) concur that learners should be encouraged through teaching "to think hard and to modify their thinking, rather than teach empirical procedures that do not take their thinking into account". Similarly Battista (2006, p. 140) talks of a "major challenge in teaching to [help] students make genuine sense of mathematical ideas".

Teaching, involving the use of different measurement tools, generally tends to be very limited and superficial. In this regard Osborne (1980, p. 56) states that "without a sense of the nature of the relationship between these tools and what is being measured, little insight into the scientific and mathematical process is obtained". This can also be ascribed to "insufficient experience with preliminary concepts" and insufficient "opportunity to tie together a base of geometric understanding with actual number concepts of measurement". This is possibly manifested in learners' inability to manipulate numbers, and to meaningfully convert from one unit of measurement to another, be it related to mass, capacity or length.

Theories that inform this critique

My stance with respect to this approach to the learning and teaching of the topic under discussion is adequately supported by one of the classical learning theories, Gestalt, as to how learners order and rearrange concepts mentally to make better sense. Furthermore, Piaget's ideas and findings about learners' use of previous knowledge to enhance assimilation and conceptualisation of new concepts are revisited. Thirdly, Skemp (2006) differentiates between instrumental and relational understanding of mathematical concepts which highlights the difference between rote learning and learning for understanding by focusing on underlying relationships.

Gestalt theorists were fascinated by the way the human mind "perceives wholes out of incomplete elements" (Skaalid, 1999, p. 1). According to the Gestaltists, things or objects are affected by where they are, that is, their position in space and by what surrounds them. Consequently, these objects are more appropriately described as much "more than the sum of their parts" (Behrens, 1984, p. 49). This implies that context was considered to be very important as far as perception is concerned.

Of the six laws of perception advanced by Gestaltists, the one that deals with 'closure', seems to be important here. This law states that the human mind, in the case of incomplete shapes, experience[s] these shapes as incomplete and compels "the learner to want to discover what's missing, rather than concentrating on the given instruction" (Dabbagh, 1999, p. 3). Here I want to refer to the gap that is created between, for example, meter and centimeter, by the omission of decimeter in the curriculum with respect to the SI system. Another principle, namely that of 'similarity' states that "things which share visual characteristics such as shape, size, color, texture, value or orientation" (Skaalid, 1999, p. 3) will be perceived as belonging together. In this case it is obvious, for example, that the 1000 in the decimal number system corresponds to kilometre as 1000 metres in the SI system.

The Gestalt theorists were mainly interested in "the entirety (the whole) of the problem or experience. To Wertheimer [a Gestaltist], truth was determined by the entire structure of experience rather than by individual sensation or perceptions." (Dabbagh, 1999, p. 5). In this regard I think of "broken up" bits of knowledge such as 1000 mm = 1 m, 100 cm = 1 m, etc. that learners are expected to memorize and apply in order to do conversions.

Piaget's ideas of assimilation and accommodation were mentioned earlier on. At this stage I briefly want to touch on the meaning of these terms together with his use of the concepts 'schema' and 'equilibrium'. Assimilation takes place when the learner perceives new phenomena or occurrences in terms of existing schemas (mental building blocks). Accommodation, which "refers to the process of changing internal mental structures to provide consistency with external reality", "occurs when existing schemas or operations must be modified or new schemas are created to account for a new experience" (Bhattacharya and Han, 2001, p. 2). The internal process of a learner making sense of external occurrences according to his or her internal experiences in order to achieve a balance between assimilation and accommodation is referred to as 'equilibrium'.

It is my contention that by allowing learners to work with the SI units in relation to the decimal number system accommodation and assimilation are facilitated and that a state of equilibrium is reached sooner. This approach emphasises and highlights the link between, and integration of the two systems

The issue of fragmentation or compartmentalisation also links up with Skemp's (2006) differentiation between instrumental and relational understanding of teaching mathematics. Instrumental understanding methods relate to the rote learning of rules, laws, procedures and algorithms, that is, learners overwhelmingly depend on guidance for learning new methods or techniques through knowledge transmission. Relational understanding methods relate to understanding relationships and connections between phenomena, concepts or mathematical ideas, allowing learners insight and knowledge as to what to do and why. Skemp (2006, p. 95) maintains that "learning relational mathematics consists of building up a conceptual structure (schema) from which its possessor can ... produce an unlimited number of plans for getting from any starting point within his schema to any finishing point".

Problems relating to conversions

By isolating or compartmentalising certain SI units, the curriculum developers may be denying the learner the chance to explore, make better sense of and get a better grip on the relationships that exist among the different SI (metric) units and the decimal (denary) system. Coherence in a sense is thus forfeited. At the same time one wonders what the reasons or rationale might be for approaching measurement in this manner. Is it related to the notion that "relational understanding would take too long to achieve" or that the relational understanding of this "particular topic is too difficult" (Skemp, 2006, p. 93)?

The problems that both learners and teachers encounter in this regard seem to be related to their sense of, "the nature of the relationship" (Osborne, 1980, p. 56) between different measuring units of length for example. Kamii (2006, p. 155) maintains that learners often did not grasp units of measurement which may relate to how estimation is taught and the kind of estimation activities used to develop measurement concepts.

For some reason the value of estimation to develop, “reinforce and elaborate the fundamental structural concepts of measurement” (Osborne, 1980, p. 55) may be under-valued.

The value of estimation is generally considered to be crucial for concept development. Osborne (1980, p. 61) maintains that “estimation skills require constant and frequent practice or they evaporate”. He also states that although “any estimate is correct in an absolute sense”, “the premium must be on which estimate is better”. Activities that allow learners the opportunity to compare their estimates with actual accurate measurement should be built into the process of concept development. These estimation activities should be applied with respect to length, mass, volume, temperature, etc. This would ensure that estimation is used in a variety of situations, thereby enhancing learners’ understanding of its usefulness (Osborne, 1980, p. 66).

Asking learners to show or indicate different lengths by using their fingers, arms or feet reflected that they generally had a limited sense of the meaning of length or distance. A typical activity (see Table 1 below) for estimating distance or mass or capacity / volume estimation exercises could look as follows:

object	Estimation of length or capacity or mass	actual measurement	difference

Table 1: Estimation

A practical way of counteracting difficulties with conversion

Some pertinent questions to pose are the following: What are the differences between our number system and the SI system? What are the similarities between our decimal number system and the SI system? Why not use these similarities as a foundation from which to expand understanding of the different measuring units? Integration of concepts would facilitate understanding (Kheong, 1997), as well as the use constructs to enhance conceptualisation, and not fall back on compelling learners to memorise statements such as 100 cm = 1m, 1000 mm = 1m, etc. without learners fully realising the meaning thereof.

Teaching conversion in context, that is seeing all parts in relation to one another, rather than working with numerical values in isolation, makes much more sense to learners. Learners have difficulty making the connection when dealing with concepts out of context. What does it mean to convert 1 m to cm? Would a grade 4, 5 or 6 learner be able to say that the 1 m is cut up into a 100 little lengths of 1cm each? It would make more sense to do this in terms of a physical object such as a metre long plank or piece of rope or string. What does it mean to convert 0,8 g to mg? Does it mean to the learner, dividing the 0,8g up into 1000 small quantities, in this case milligrams (mg)? Does it mean how many full milligram quantities can you get out of 0,8g? Or does it mean magically changing the quantity to something a thousand times ‘bigger’ or ‘smaller’, referred to as “unit iteration (Kamii & Clark, 1997, p. 120)?

In the NCS (DBE, 2011, p. 25) the specific content on length is indicated as follows:

> Grade 4: Conversions include converting between millimetres (mm), and centimetres (cm); centimetres (cm) and metres (m); metres (m) and kilometres (km). Conversions limited to whole numbers and common fractions
>
> Grade 5: Conversions include converting between any of the following units: millimetres (mm); centimetres (cm); metres (m) and kilometres (km)
>
> Conversions limited to whole numbers and common fractions
>
> Grade 6: Conversions include converting between any of the following units: millimetres (mm); centimetres (cm); metres (m); kilometres (km). Conversions should include common fraction and decimal fractions to 2 decimal places

The "list above" only contains the following: "length using millimetres (mm), centimetres (cm), metres (m) and kilometres (km). Did the curriculum developers anticipate that to include the other units of length would hamper conceptualisation, or not have the "'logico-mathematical knowledge" to make mental relationships (Kamii, 2006, p. 158)? Why for instance is 'decimetre' not mentioned in the curriculum statement, taking into consideration its close relationship with volume ($1dm^3 = 1$ litre)? Furthermore, the term or concept *hectometre* is not mentioned anywhere in the NCS or RNCS (Revised National Curriculum Statement), yet calculations involving *hectare* appeared in one of the past CTA's (Common Task for Assessment, General Education and Training).

Before proceeding to the use of the conversion table it needs to be emphasised that to deal with conversions successfully also implies an understanding of the nature of the SI measurement system. According to Osborne (1980, p. 56) such an understanding of the characteristics of the metric system means that the learner "possesses an ideational structure that can serve as a base both for problem solving and for measurement ideas". This structure according to Osborne (1980, p. 56-57) includes the following:

> 1. Number assignment: To measure an object is to assign a number to an attribute or a state of the object…
>
> 2. Comparison: If object A is "contained by" Object B, then the measure of object A is less than the measure of object B…
>
> 3. Congruence: If object A and object B are congruent, then the numbers that are their measures are the same.
>
> 4. Units with iteration: There is a special object or state in any measurement system to which the number one is assigned [such as metre to measure distance in the SI system]. The object with measure one is the base for identifying the functional rules that determine the number assignment for any object or state…
>
> 5. "Additivity…" (addition of two or more lengths).

In this discussion I make use of the ‘conversion’ table below as an attempt to explain a particular approach of dealing with conversion of one SI unit to another. The table should be seen and used as a tool, especially with learners who experience difficulty converting from one unit of measurement to another. It is an auxiliary tool to help learners see a particular value in context and in relation to another, for example 0,002 km as opposed to 2 meters. It needs to be emphasised that all values used in the table below should be read in relation to the meter as unit of measurement. A ‘typical’ conversion table could look like this:

←	**Thousands** **1000 (10^3)**	**Hundreds** **100 (10^2)**	**Tens** **10 (10^1)**	**Units** **1 (10^0)**	**Tenths** **$^1/_{10}$ (10^{-1})**	**Hundredths** **$^1/_{100}$ (10^{-2})**	**Thousandths** **$^1/_{1000}$ (10^{-3})**	→
LARGER QUANTITIES OR DISTANCES mega meter	kilo-metre	hecto-metre	deca-metre	metre	deci-metre	centi-metre	milli-metre	SMALLER QUANTITIES OR DISTANCES nanometre
	kilo-gram	hecto-gram	deca-gram	gram	deci-gram	centi-gram	milli-gram	
	kilo-litre	hecto-litre	deca-litre	litre	deci-litre	centi-litre	milli-litre	
				5	*0*	*0cm*		
				5	0	0	0mm	
	0,	0	0	5km				
				5	0dm			
	7km							
	7	0	0	0 m				
	7	0	0	0	0	0cm		

Table 2: Distance/mass/volume

Using the above-mentioned (Table 2) makes it quite easy to convert 5m for example to centimetres, millimetres or to write it as a decimal fraction of a kilometre. From Table 2 it is evidently much clearer to follow what actually happens. It is easy to see the correspondence between the decimal (denary) number system and the SI system. It is evident that the bigger picture is much more than the mere sum of the constituent parts. By leaving out or ignoring for instance the decimetre, hectometre or decametre as units of measurement gaps or voids are created that may cause problems for learners. Starting with the Gestalt (the whole), and allowing learners to perceive the interrelationships and inter-connectedness of the different measuring units, I maintain would be experienced as much more meaningful and enhance assimilation.

From the table, using metre as point of reference, it is also easy to observe that 'kilo' means 'thousand', 'hecto' means 'hundred', 'deci' means 'tenth', 'centi' means 'hundredth' and 'milli' means 'thousandth' in terms of for instance the metre as SI unit to measure distance. This means that the position of each is important and indicates its value in relation to others. The table could be altered to include only information that relates to capacity or length or mass when dealing with specifics for practical reasons. The table as it appears above could thus be adapted according to the level of the learner.

Does the curriculum really advocate fragmentation and compartmentalisation of mathematics content by dealing with SI units in the following manner:1 metre = 1000 mm or 1mm = 0,001 m, etc.? One gets the sense that the curriculum in this regard advocates an instrumental approach to learning mathematical concepts. Learners are supposed to learn and know this by heart – and not necessarily with insight and understanding. By isolating the units and dealing with them as separate entities makes it more difficult for learners to comprehend. Teaching focuses more on "establishing familiarity with and use of the metric units, than in using learning about metric measurement to reveal the nature of measurement" (Osborne, 1980, p. 56). Consequently, learners may end up "with several particulate bits of discrete knowledge about measurement rather than developing a feel for and an understanding about the nature of measurement as a system".

Simultaneously dealing with decimal fractions

A contentious issue is that of 'shifting' the comma when converting from one unit to the other. Does the comma shift? Or is it just a convenient way of explaining what happens when converting? Dealing with conversions as suggested also creates the opportunity to address decimal fractions in a much more meaningful way - that is, in context and therefore much more realistic. Telling learners to divide a 0,8 metre length of wood into 5 equal pieces, makes much more sense than merely telling learners to divide 0,8 by 5.

From the Table 2 one could easily determine what decimal fraction 5 metres is of a kilometre, or of a hectometre. Similarly centimetres, decimetres or millimetres could be written as a fraction of a metre. Simple addition and subtraction manipulations could also be done using a table like this. Adding or subtracting quantities can be done using the conversion table (see Table 3). For example to add 45 g, 2 876 cg and 8 943mg could pose problems when learners try to add the values or quantities either by writing them underneath one another or next to one another in the same line. In this case the answer can either be written as 82,703 g or 8270,3 cg or 82703 mg or even as 0,082703 kg as the quantity "is being conserved" (Hiebert, 1981, p. 208). This means that the teacher must give specific instructions and tell learners whether the answer should be given in grams, centigrams or milligrams, or kilograms. The following table can be used to accurately position the various quantities:

←	**Thousands** **1000 (10^3)**	**Hundreds** **100 (10^2)**	**Tens** **10 (10^1)**	**Units** **1 (10^0)**	**Tenths** **$^1/_{10}$ (10^{-1})**	**Hundredths** **$^1/_{100}$ (10^{-2})**	**Thousandths** **$^1/_{1000}$ (10^{-3})**	→
LARGER QUANTITIES	kilo-gram	hecto-gram	deca-gram	gram	deci-gram	centi-gram	milli-gram	SMALLER QUANTITIES
			4	5g				
			2	8	7	6cg		
		+		8	9	4	3mg	
			8	**2**	**7**	**0**	**3**	

Table 3: Mass

The role of actual measurement

The role of actual measurement, that is to physically explore the relationship between SI units, is essential before working with them theoretically as suggested with the unit table. None of this can occur if learners were not exposed to actually measuring and comparing the different units of length for example. They should practically count how many millimetres there are in a centimetre, centimetres in a decimetre, decimetres in a metre, etc.

Learners have to be taken outside to walk 10, or 100 or 1000 metres to help them conceptualize the magnitude of decametre, hectometre, etc. This very seldom happens inside the real classroom. This state of affairs was confirmed by a few hundred intermediate phase teachers who went through CTLI training during the course of 2005, 2007 and 2008. The teacher should thus create opportunities for contextualisation with SI units, as well as much greater integration with learning areas such as Geography and Natural Sciences.

CONCLUDING COMMENTS

Based on my experience as facilitator and trainer, teachers tend to shy away from teaching topics or aspects of mathematics that they do not understand or that they have limited knowledge of. This is also true in the case of conversion. Many of the 200 teachers at the CTLI (2005) who attended my workshops on measurement, claimed that they had a much better understanding of conversion and that they would apply a practical approach (Hiebert, 1984) in their classrooms. It is interesting that many of them never heard of decimetre or hectometre before. They claimed that this holistic approach facilitated their understanding and assisted them in converting quantities quicker and more accurately.

It is important to realise that statements such as 1000 m = 1 km, 100 cm = 1 m, etc. can still be learned by heart, but only once learners have a good idea of the bigger picture as discussed earlier. This would allow them to understand that 1000 metres is exactly the same 1 kilometre, since *kilo*- means *thousand*, for instance. A deliberate effort should thus be made to help learners understand and learn the meanings of all the Latin prefixes such as *kilo-, hecto-, deca-, deci-, centi- and milli- in* terms of the concept metre.

The use of Table 2 or variations thereof should be viewed as a tool to help learners see relationships and develop insight into what happens when converting from one unit of measurement to another. Its value is especially evident when introducing conversions. Once learners have gained insight, the use thereof may be discontinued or it can be used as a means of reference.

It is apparent that an instrumental approach to the learning of mathematics necessitates a teacher-centred approach of transmitting rules and procedures which is limiting and less adaptable to new tasks (Skemp, 2006; Bragg & Outhred, 2004). In contrast, teaching for relational understanding requires a more learner-centred approach by giving learners scope and allowing them opportunities to explore relationships, consequently fostering independent thinking, and enhancing understanding and remembering.

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